On the second lowest Hamming weight of binary projective Reed-Muller codes

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In coding theory one is not only interested in the minimum distance (= least Hamming weight of a nonzero codeword) but also in higher Hamming weights to study the code performance. In the case of GRM codes this question also has algebraic-geometric interpretations.

The determination of the second lowest Hamming weight $W^{(2)}_{\mathrm{GRM}}(n, d)$ for all d was completed only in 2010, and $W^{(2)}_{\mathrm{GRM}}(n, d) = (q-b)q^{n-a-1} + cq^{n-a-2}$ where c is equal to $b-1$, $q-1$ or q, according to the values of q and d.

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The dimension of $\mathrm{GRM}(n, d)$ is well known as well as its minimum distance $W^{(1)}_{\mathrm{GRM}}(n, d)$. If $d \geq n(q-1)$ then $\mathrm{GRM}(n, d) = \mathbb{F}_q^{q^n}$ and $W^{(1)}_{\mathrm{GRM}}(n, d) = 1$. To find the minimum distance for $1 \leq d \leq n(q-1)$ write d uniquely as $d = a(q-1) + b$ with $0 < b \leq q-1$ and then $W^{(1)}_{\mathrm{GRM}}(n, d) = (q-b)q^{n-a-1}$.

In coding theory one is not only interested in the minimum distance (= least Hamming weight of a nonzero codeword) but also in higher Hamming weights to study the code performance. In the case of GRM codes this question also has algebraic-geometric interpretations.

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S. Ballet, R. Rolland, On low weight codewords of generalized affine and projective Reed-Muller codes. Des. Codes Cryptogr. **73**(2) (2014) 271–297.

On the present work, joint with Victor G.L. Neumann, we determined all the next-to-minimal weights $W^{(2)}_{\mathrm{PRM}}(n, d)$ for PRM codes defined over $\mathbb{F}_2$. We used a geometric approach to the problem, adapting and modifying methods we found in the thesis of D. Erickson.

An upper bound for $W^{(2)}_{\mathrm{PRM}}(n, d)$

From Serre + Sørensen we know that $W^{(1)}_{\mathrm{PRM}}(n, d) = W^{(1)}_{\mathrm{GRM}}(n, d-1)$ so our idea was to investigate if $W^{(2)}_{\mathrm{PRM}}(n, d) = W^{(2)}_{\mathrm{GRM}}(n, d-1)$.

Observe that if $g \in \mathbb{F}_q[X_1, \ldots, X_n]$ with $\deg(g) = d - 1$ and we set $g^{(h)} \in \mathbb{F}_q[X_0, \ldots, X_n]$ to be the homogenization of g with respect to X_0 then $\deg(X_0 g^{(h)}) = d$ and the weight of the codeword $\psi(X_0 g^{(h)}) = (X_0 g^{(h)}(Q_1), \ldots, X_0 g^{(h)}(Q_N)) \in \mathrm{PRM}(n, d)$ is the same of the codeword $\varphi(g) = (g(P_1), \ldots, g(P_{q^n})) \in \mathrm{GRM}(n, d-1)$ hence

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Our aim was to show that there is no $f \in \mathbb{F}_q[X_0, \ldots, X_n]_d$ such that the weight of $\psi(f)$ lies in the interval $(W^{(1)}_{\mathrm{PRM}}(n, d), W^{(2)}_{\mathrm{GRM}}(n, d-1))$.

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Results of geometrical nature

Definition. Let $f \in \mathbb{F}_q[X_0, \ldots, X_n]_d$, the set of points of $\mathbb{P}^n(\mathbb{F}_q)$ which are not zeros of f is called the support of f, and we denote its cardinality by $|f|$, hence $|f|$ is the weight of $\psi(f)$.

An important property of the support is the following.

Lemma 1. Let $f \in \mathbb{F}_q[X_0, \ldots, X_n]_d$ be a nonzero polynomial, and let S be its support. Let $G \subset \mathbb{P}^n(\mathbb{F}_q)$ be a linear subspace of dimension s, with $s \in \{1, \ldots, n-1\}$, then either $S \cap G = \emptyset$ or $|S \cap G| \geq W^{(1)}_{\mathrm{PRM}}(s, d)$.

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Lemma 2. Let $f \in \mathbb{F}_q[X_0, \ldots, X_n]_d$ be a polynomial with a nonempty support S. If there exists a hyperplane $H \subset \mathbb{P}^n(\mathbb{F}_q)$ such that $S \cap H = \emptyset$ then $|f| = W^{(1)}_{\mathrm{PRM}}(n, d)$ or $|f| \geq W^{(2)}_{\mathrm{GRM}}(n, d-1)$. (In other words, either $|f| = W^{(1)}_{\mathrm{PRM}}(n, d)$ or $|f| \notin (W^{(1)}_{\mathrm{PRM}}(n, d), W^{(2)}_{\mathrm{GRM}}(n, d-1)$.)

The next result gives conditions for a subset of $\mathbb{P}^n(\mathbb{F}_q)$ to miss a hyperplane.

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Before we present the main result, we make remarks specific to the case where $q = 2$.

We are writing $d - 1 = k(q-1) + \ell$ with $0 < \ell \leq q - 1$ so when $q = 2$ we get that $\ell = 1$ and $d - 1 = k + 1$. Also, we are assuming that $2 \leq d \leq n(q-1)$, so now we have $2 \leq d \leq n$, and in particular $n \geq 2$.

From the table with the values for $W^{(2)}_{\mathrm{GRM}}(n, d-1)$ we get that

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(Parts of the) Proof. Let $k = 0$ and $n \geq 3$, and take $f \in \mathbb{F}_q[X_0, \ldots, X_n]_d$, $f \neq 0$, such that $0 \neq |f| < (1 + \frac{1}{2})$

Let S be the support of f. From Lemma 1 we get that if $G \subset \mathbb{P}^n(\mathbb{F}_q)$ is a linear subspace of dimension s, with $s \in \{1, \ldots, n - 1\}$, then either $S \cap G = \emptyset$ or $|S \cap G| \geq W^{(1)}_{\mathrm{PRM}}(s, d)$. From Lemma 3, since $|f| < (1 + \frac{1}{q})(q - \ell)q^{n-k-1}$ we get that $|f| = W^{(1)}_{\mathrm{PRM}}(n, d)$ or $|f| \geq W^{(2)}_{\mathrm{GRM}}(n, d - 1)$. This proves that $3 \cdot 2^{n-2} \leq W^{(2)}_{\mathrm{PRM}}(n, d)$. Taking $g = X_0X_3 + X_1X_2$ one may show that $|g| = 3 \cdot 2^{n-2}$.

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Another question that arises from the study of Hamming weights of GRM and PRM codes is the following:
assume that $f \in \mathbb{F}_q[X_1, \ldots, X_n]$, with $\deg(f) \leq d$, and $g \in \mathbb{F}_q[X_0, \ldots, X_n]_d$ are such that $\varphi(f) \in \mathrm{GRM}(n, d)$ and $\psi(f) \in \mathrm{PRM}(n, d)$ are minimal weight codewords. Can we conclude that f or g have a special form?
The answer is yes in both cases, and in both cases the polynomials must be a specific product of degree one polynomials - see Delsarte, Goethals and Mac Williams (1970) for the case of GRM codes, and Rolland (2010) for PRM codes. These answers have geometrical implications.

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