Semiotic-Cognitive Theory of Learning

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Abstract: On the basis of Vygotsky's definition of conceptualization (1995), this work links Piaget and García's stages of development through their triads (1984) to the semiotics introduced by Peirce (1974), who classifies signs in three categories which are associated with three inferences: abduction, induction, and deduction. As a result, the construction of knowledge begins with an abduction on the first stage, originating from a result which is presented (either deliberately or not, when teaching) as a problem for the subject and which destabilizes their Interpretive System. The relations arising from abductive generalization are established on the second stage as an interaction between form and content. By inductive or completive generalization, the form becomes deductive on the third stage. It is detached from all content, and is incorporated and linked to the rest of the forms of the Interpretive System. From the semiotic point of view, this implies the following progression: icon → index → symbol.

1. Introduction
The semiotic-cognitive learning theory, which is presented in this work, is a theory of knowledge acquisition that involves central aspects of the theories developed by Jean Piaget and Rolando García (Piaget & García, 1984; Piaget, 2002; García, 2000), Lev Vygotsky (1995), and Charles Peirce (1974). It combines them in a formulation which, on the basis of Vygotsky's definition of conceptualization, states a correspondence among the three stages of cognitive development introduced by Piaget and García, and the three sign classification categories together with the three forms of inference proposed by Peirce (González, 2012). It is strictly focused on learning from the acquisition of hypothetical-deductive thinking, which is assumed as already acquired in secondary and higher education.

2. Development
Figure 1 shows the main aspects that set the foundations of a conceptualization process in the mentioned schools of thought. Vygotsky regards it as a generalization process in a system of concepts which evolves in a line of generality. Along this line, a concept acquires a degree of generality. Relationships of generality are established between concepts. Conceptual acquisition is a socio-historical-cultural process which first appears in an interpsychological level among people and is then internalized in an intrapsychological level (Vygotsky, 1995).

In Psychogenesis and the History of Science, Piaget and García also conceive the idea of conceptualization as a generalization process. It is divided in three stages: intra, inter, and trans, as mechanisms that are respectively focused on the concept attributes or conceptual object (CO), the relations between concepts, and the structure formed by these relations defined as transformations.

A typical example given by these authors is the case of plane geometric figures. A triangle, for instance, is defined by its attributes, but it can be regarded as translation and rotation invariant.
These relations or transformations among points of the plane do not alter the distance among the points of the triangle. In turn, they form a group structure. Nevertheless, despite defining three stages, the authors consider only two types of generalization: *inductive* generalization and *completive* (or *constructive*) generalization. *Inductive* generalization consists in generalizing the *result*. Thus, a relation applied to some cases is applied to all of the cases in a determined *conceptual system*. The latter occurs when these *relations* that are associated with a result become *necessary* and the *result* becomes a *necessary condition* for the *relations*. Therefore, given a *case*, the *relations* necessarily lead to the result. In the *trans* stage, the different relations are *coordinated* in the so-called *Interpretive System* (*IS*). In addition, Piaget (García, 2000) states that the *CO* is *assimilated* by the *IS* and transformed by it. In turn, the *IS* *adjusts* to the *CO* and is simultaneously transformed in a global process of *cognitive equilibrium*.

![Diagram of the main components of conceptualization](image)

Vygostsky initially considers the sign as an *instrument of conceptualization* and Piaget proposes a *semiotic function* that surpasses the natural language (Radford, 2006). But the *sign* is regarded here as the *bearer of meaning*, i.e., of the *concept*. Taking Vygotsky's theory as a basis, it can be said that
in the same way a concept is a generalization in a system of concepts, a *sign* is a *generalization in a system of signs*. Conceptualizing means generalizing, but also simultaneously defining a system of signs. There is not one without the other. That is, from the point of view of its meaning, the sign depends on the system of signs it is *interrelated* with. These established relationships between signs are then relationships between concepts, and hence *relationships of generality*, as defined by Vygotsky.

For instance, the concept of integer in the number system and its operations can be defined from the generalization of the concept of subtraction among natural numbers for the cases that cannot be solved in this set (ex. 5-9). The set of integers is also a particular case for the set of rational numbers when the denominator is 1. Thus, it is a *particular case* of the relationship of generality among integers that defines rational numbers \((a.d=b.c \text{ in } a/b= c/d \text{ with integers } a,b,c,d)\). We can see that the signs used with the integers are the ones that will define the signs used with the rational numbers through the relationships of generality that interrelate them. On the other hand, this system of signs implies a *semiotic context* that is given by the *set of relationships of generality* that are established among the *signs* of the system. If, when interpreting these relationships, they are univocally defined (e.g., when they are conventions in a specific field, such as \(Z\) for integers, SOS meaning 'help', traffic lights, etc.), there will be a *syntagmatic context*. If, on the other hand, the relationships of generality that can be established among signs are open to the interpreter's different possibilities of interpretation, there will be a *paradigmatic context*. These definitions of *concept* and *sign* imply, therefore, a prior *system of concepts (of signs)* that is necessary to construct them. As Rolando García (2000) proposes, they can be considered *interpreted data*, what he denominates *observable* (a concept that is taken from physics).

Charles Peirce (1974) provides both a *theory of signs* and a *system of correlative inferences*. First, Peirce conceives the *sign* in its aspect of *representamen* as a bearer of a *quality* which stands in place of something else. It represents something – its *object*, a *sign* of real existence, which is in turn interpreted by someone by means of another *sign* denominated *interpretant*. For example, \(H_2O\) is an *interpretant* (one of the many that are possible) of the *representamen* ‘water’ (word) which denotes an *object* that is a *sign* when it is related to other objects (location) and bears the qualities of *colourless, odourless, and tasteless liquid*. Any physical object is an *indexical* sign that can only be conceptualized from a system of signs, and therefore requires previous conceptualizations (or signs). Moreover, new conceptualizations will require new signs.

Peirce classifies signs into three categories denominated *firstness, secondness and thirdness*. Firstness implies the *quality*, i.e., the *attributes* that are inherent to the object, *abstracting* it from the reference to another object. *Secondness* involves taking the object that bears the attribute in *relation* to another object. In this case, Peirce attributes an *existence* to the object as an *indexical*
sign (the object that really exists and bears the quality). Thirdness is introduced by a sign, the interpretant, which is a law defined by the relations introduced in secondness. Nevertheless, these concepts are relative. For example, an interpretant of a given degree of generality can become a representamen in the following stage. Therefore, it is possible to find each of these signs – representamen, object, and interpretant – in all three categories.

The signs defined by Peirce are shown in table 1 (Peirce, 1974; Vitale, 2002; Marafiotti, 2002; Magariños de Morentin, 2008). They are classified by their function. Following what was previously described, we can see that their classification is carried out according to their degree of generality. A qualisign is a quality (for instance, color) embedded in the sinsign (traffic light) that expresses a legisign (a law: the red light means you cannot cross the street). In turn, an icon is a sign (object) that is analogous to another object (that is, an attribute that is common to the objects, such as a color, or an image that evokes an object, such as a picture, the points in common between two different theories, etc.). An index is a sign (object) of real existence and contiguity that attracts our attention towards an object (an arrow \( \rightarrow \) among natural numbers: \( 2 \rightarrow 4 \), 2 is assigned 4). A symbol is a sign (object) that expresses a level of generality by means of a law (for instance, the variables \( n \in N \), \( m \in N \), and the expression that is based on them: \( m=2n \)). A symbol is an object in the third category of generalization or abstraction, and therefore, an interpretant.

A rheme is a sign that represents a certain kind of objects (e.g., a flower) and refers to qualities. Thus, it is a firstness. A dicisign is a proposition that involves rhemes. It is a secondness, so it implies both a relation and an object of real existence. An argument is a form of reasoning that involves a dicisign as a premise and another dicisign as a conclusion. It is, in essence, an interpretant.

Table 1: Peirce's classification of signs

<table>
<thead>
<tr>
<th></th>
<th>FIRSTNESS</th>
<th>SECONDNESS</th>
<th>THIRDNESS</th>
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<tbody>
<tr>
<td>REPRESENTAMEN</td>
<td>qualisign</td>
<td>sinsign</td>
<td>legisign</td>
</tr>
<tr>
<td>OBJECT</td>
<td>icon</td>
<td>index</td>
<td>symbol</td>
</tr>
<tr>
<td>INTERPRETANT</td>
<td>rheme</td>
<td>dicisign</td>
<td>argument</td>
</tr>
</tbody>
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Arrows indicate the direction in which the degree of generality increases.

Nevertheless, this categorization depends on the level of generality. An interpretant in a determined level of generality can become an object in the next level, and even go through all three categories.
in that same level. Therefore, they constitute fractal relationships which are, as such, also dialectical (García, 2000; Piaget, 2002).

An example of a sign going through all three categories in the same level can be found in the natural numbers: their central property is the existence of a consecutive. A succession of similar objects (e.g., balls) constitutes an iconic representation, and its representamens are 1, 2, 3,.... We can anticipate the need of a generic representamen, a symbol, which will be a variable. But, for example, in order for $m$ to become a variable, a previous step is required. Each number must become a possible result of the variable by means of an equivalence relation, i.e., $m=1$, $m=2$, $m=3$.... In this case the variable acts as an index, since it will indicate a specific number. Here, the sign $m$ is affected by the object (the given natural number) when the equivalence relation is established. This shows, on the other hand, the existence of a relationship inherent to secondness. It also constitutes an interaction form (the symbol $m$) - content (the specific number). This relationship applies to all the natural numbers from the equivalence relation of the variable with any natural number. But the generalization of this operation to all the natural numbers will become a symbol when representing it by means of a variable becomes a necessary condition.

The characteristics of Piaget and García's stages intra, inter, trans (IaIrT) and the categories of firstness, secondness, and thirdness introduced by Peirce lead to the correspondence intra $\leftrightarrow$ firstness, inter $\leftrightarrow$ secondness, trans $\leftrightarrow$ thirdness, which is proposed in this work. The distribution of signs in the three categories is the semiotic expression of the cognitive mechanisms (IaIrT).

Piaget and García propose two forms of generalization: inductive (or empirical) generalization and constructive (or completive) generalization (García, 2000). The former involves an empirical abstraction of determined relationships based on attributes verified in an empirical object, which, in some cases, if repeated, applies to the set of objects under consideration as well. The latter involves a reflective abstraction that projects all the inferred relationships in a superior level of coordination that makes them deductive (which means that these relationships must become necessary). Piaget and García find a correlation between inductive generalization and the intra phase because it deals with the object attributes, and propose another correlation between completive generalization and the inter phase, which deals with relations.

In addition to induction and deduction (which correspond to firstness and secondness), Peirce's semiotics introduce another type of inference, which he denominates abduction. Abduction corresponds to firstness, which was not considered by Piaget and García, even though Piaget (Piaget & García, 1997; Hernández Ulloa, 2008) mentions it in his later years as an element to be considered. On the other hand, there is a consequence of the correlation between the IaIrT triads
With regard to the attributes of the CO, abduction must be taken into account in the situation designated as cases. In the second phase, induction must be taken into account. Therefore, the first phase is the contents phase (in Piagetian terms, forms will be constructed in relation to these contents) and their attributes, which were already constructed in previous stages. The correlation between categories and mechanisms proposed in this work constitutes the base of a clear semiotic expression of these mechanisms, which are represented by their inferences. The passage from one phase to another will be achieved by means of generalizations (since they result in a succession of abstractions that increase the degree of generality). Here, these generalizations will be abductive, inductive and completive. The last two will lead to the third phase, where deduction is reached as a third inference. The semiotic definition of these inferences help us better establish the correlations presented and the role of the signs in each of them.

From the semiotic point of view, the definition of these inferences (Fig.1) is founded on the concepts of case, result, and rule corresponding to firstness, secondness, and thirdness, respectively. The following is a classic example: there are bags containing little balls of different colors (each bag is a case), little balls of different colors taken from one of those bags on a table (result), bags containing little balls of a same color (rule). The case involves an attribute (color), the result involves a relationship (between the little balls with certain colors on the table and the bag they were taken from), and the rule involves a structure (the balls of a same color in a same bag). Based on these elements, we can state the following (Marafioti, 2002; Vitale, 2002):

**Deduction**: all the balls in this bag are white (rule). These balls were taken from this bag (case), therefore (with certainty) these balls are white (result). There is a rule from which, given a case, a result can be inferred.

**Induction**: these balls were taken from this bag (case), these balls are white (result), therefore (probably) all the balls in this bag are white (rule). Given a case and a result, a rule can be inferred.

**Abduction**: all the balls in this bag are white (rule), these balls are white (result), therefore (probably) these balls were taken from this bag (case). Given a rule and a result, a case can be inferred.

The complete incorporation of a given CO requires the passage through the three stages or phases which are the base of the cognitive mechanisms, that is, of the three inferences expressed in their corresponding signs. In fact, as previously stated, abduction is based on attributes by means of iconicity, since it expresses analogies between different objects, such as the whiteness of the balls in the bag and the ones on the table. Induction is based on indexicality, since the result is expressed by means of an indexical relationship between objects of real existence, such as the bags containing balls and the balls on the table (a relationship which is based on attributes, as is 'white').

The conclusion is drawn by means of this indexical relationship. Finally, on the basis of these
attributes and relationships, deduction is expressed with a symbol, since it corresponds to a general law. In the above example, the law is a rule that establishes that all the balls in a given bag are white. Then, perforce, if we take some balls from that bag (case) and put them on a table, these balls will be white (result). Here, the concept of logical necessity comes into play.

The passage of the CO through the three stages is the process that transforms its aspects. They change from those of an iconic sign to those of a symbolic sign when it is incorporated as an interpretant in the Interpretive System. How is this passage produced? In the first phase, the cases refer to the contents of the CO. These will have determined attributes which will, in turn, define a system of contents (e.g., the balls in the bags, the system of natural numbers, etc.). Abduction requires a result that is the trigger/motivator of new knowledge and destabilizes the IS. Peirce regarded this fact (result) as surprising or exceptional. Nevertheless, if it cannot be incorporated by the IS, it will be destabilized. This specific result involves determined contents defined in this phase and a relationship to be revealed as a hypothesis by means of abduction. The process is initiated with the genesis of the form developed from the stabilized IS (before the result destabilizes it) and it will involve the rule which, together with the result, is part of the definition of abduction. Therefore, this rule (here, the hypothesized relationship) will be in function of the case (the contents attributes) based on the result. Thus, the case will be inferred by the rule and the result, as is required by abduction. Pythagorean triples are an example of this (González, 2012). When considering the triples of natural numbers (3, 4, 5) and (6, 8, 10), where the components fulfill \(3^2 + 4^2 = 5^2\) and \(6^2 + 8^2 = 10^2\), some questions about the obtention of all the Pythagorean triples of natural numbers arise: how many are there? Which ones are they? How can they be obtained? In this case, the contents are triples of natural numbers and their attributes are those corresponding to the natural numbers and the Pythagorean relationship. Both chosen triples are connected by an indexical relationship which is quite easy to notice in this case: \((6, 8, 10) = 2.(3, 4, 5)\). This generates the proportional form applied to the triples. In order to obtain this relationship, it is necessary to compare cases. Then, given a result \((6, 8, 10)\) and the proportional form rule, it is possible to infer the case \((3, 4, 5)\) and all the other cases associated to these triples by resorting to retroduction (from the rule to the case). When a new case is presented, such as the Pythagorean triple \((5, 12, 13)\), which is not proportional to the former triples, the established rule cannot be applied, and new cases and new abductions will be possible and necessary.

The form obtained on the basis of abduction in the first phase is applied in the second phase to the cases that demonstrate and reproduce the results establishing the indexical relationship. It is usually said that abduction explains the results. Moreover, this is the phase where form and content interact. In these conditions and in this stage, the next step is exploring how the form can be applied to all the cases that constitute the system of contents defined by the attributes in the previous stage. If the
form applied to some cases can be applied to all the cases responding to similar attributes, then we can speak of inductive generalization.

In the third phase, the form obtained from the results related to the contents and their attributes becomes necessary to them, and they become a necessary condition of the form. This means that the results become deducible from the form when it is applied to all the cases. This form, which until now had an indexical expression given by its application to specific cases, acquires a symbolic character and is detached from the content, becoming a pure form. This means that it becomes a part of the interpretant, and it will be stabilized when it is coherently linked to the rest of the forms of the IS which, by incorporating it in this process of equilibration by means of assimilation and accommodation, is extended and transformed into an IS’. The following scheme is obtained by passing from the first to the third stage: \( CO \rightarrow CO' \), \( IS \rightarrow IS' \). In turn, the structure of this stage will be given by the attributes of the relationships involved.

In the example of the Pythagorean triples, the proportional form will be noted as \( n(a, b, c) \), with \( n \in \mathbb{N} \). \( (a, b, c) \) is a Pythagorean triple and is clearly a syntagmatic and symbolic form which has been detached from the initial specific contents and can be extended to any Pythagorean triple, even to other contents, such as real numbers. That is, as a form, its structure would be \( (, , ) \), which can be applied to any content when valid. If the passage from the second to the third phase is produced because the content attributes expressed by the results are necessary to obtain those forms, then a completive generalization can take place without the need of an inductive generalization. On the other hand, it is important to highlight that these are dialectic processes, and therefore these three phases are relative to a certain degree of generality. Thus, a phase that is regarded as trans in a given level can become an intra phase in the following level. Finally, when the cycle is closed, in addition to new forms, contents can be extended in the same way number sets are extended.

The semiotic bases of these three inferences show that they are collaborative and almost simultaneously formed. In fact, what we have denominated rule depends on the relations expressed by means of the results. In turn, these relations depend on the cases determined by the attributes. However, this does not mean that the passage through the three stages is simultaneous, since attributes, relations, and structures must be considered in that order. Therefore, there is an order of focus: by focusing on the attributes, relations are constructed. These relations will constitute the rule. By focusing on the relations, we can discern the structures, and by focusing on the structures, we consider the linkage of the forms in the IS.

The complete chart of correlations is as follows:
3. Application in the university entrance course

The following is an account of the previously described sequence in the specific case of university entrants.

A field study conducted at Universidad Nacional de General Sarmiento (UNGS) (González, 2012) shows that some of the students in the admission course redefine the sign $\sqrt{\cdot}$ in a diagnostic activity presented on their first day at university, based on their IS at the moment of starting the course. The students are asked to give the result of some basic and compound arithmetic operations using natural numbers. These operations are presented as written expressions ending in an equality sign, which reinforces the idea that a result is being requested. A central objective is to observe how students interpret the signs involved in these operations. Some of these involve the symbols $\sqrt{4}$ and $\sqrt{5}$. The aim is to observe how they interpret the sign $\sqrt{\cdot}$ based on the cases given. The natural numbers are the content of the cases based on their attributes. The students are requested to obtain the result of the operations $1 + \sqrt{4} = \text{ and } 1 + \sqrt{5} = \dot{\text{, where the sign equal is an index that is associated to the result, as previously stated, and induces the student to obtain it. The students who provide an answer, understand the meaning of the sign and try to obtain a result. Since $\sqrt{5}$ is an irrational number, its exact value can only be interpreted symbolically. Thus, the second expression can only be solved by means of the identity $1 + \sqrt{5} = 1 + \sqrt{5}$. This is important since it shows that the operation itself is the exact result.}

In one of the groups of students under study, 12% give the sign $\sqrt{\cdot}$ the conventional or syntagmatic meaning. They provide the following results: $1 + \sqrt{4} = 3$ and $1 + \sqrt{5} = 1 + \sqrt{5}$. In the passage attribute $\rightarrow$ relations $\rightarrow$ structure with a semiotic support: icon $\rightarrow$ index $\rightarrow$ symbol, these students show that they are in the last phase in relation to the concepts involved. These students simply infer the results by applying the conventional rule (resulting from a syntagmatic context) to the different cases. In that group, 21% of the students use the sign $\sqrt{\cdot}$ according to the conventional rule for the cases of natural numbers that are perfect squares. However, they redefine the rule in the case of
non-perfect squares: \( \sqrt{2} \rightarrow \frac{1}{2} \). In the case of 5, the answer provided is \( \sqrt{5} \rightarrow \frac{5}{2} \). That is, these students understand \( \sqrt{\cdot} \) as a division by two. This way, their results are \( 1 + \sqrt{4} = 3 \) and \( 1 + \sqrt{5} = \frac{7}{2} \).

This group is making an abduction. In fact, they focus on attributes first: the natural numbers, discriminating between perfect squares and non-perfect squares. In the first case, the conventional rule is applied. However, as previously stated, \( 1 + \sqrt{5} \) constitutes the result itself in the second case. Since the students cannot apply the conventional rule to obtain a result in the set of natural numbers (or among the numbers they know), they hypothesize the rule in function of the case. That is, they infer the case from this rule and the result. They do not consider the conventional rule (the syntagm) applicable in this case. Their semiotic context is paradigmatic. In this context, the relationships of generality are not determined because the students have not acquired the conventional rule for every case. The relationships of generality are constructed in function of the possible rules previously incorporated in their IS. Moreover, since the attributes of the case are involved, the search will be iconic. That is, by analogy, \( \sqrt{1} \rightarrow \frac{1}{2} \). The first sign is analogous to a division by 2 (since it is a square root). Therefore, these students are focused on the iconic phase. They will be able to continue to the indexical phase, but they will fail to reach the last elaboration phase of the symbol due to their ambiguous definition in function of the case. The last phase, the deductive one, will be reached when they are able to understand the fact that the symbol \( \sqrt{\cdot} \) must have an univocal signification in relation to all the cases. This will happen when they cease to depend on the cases and the form is consequently detached from the content.

Rolando García (2000) proposes a third version of the theory of equilibrium by Piaget. According to this theory, the constructive process of knowledge results from the interaction of the form (logical forms) with the content (physical world) by means of the mechanism of the IaIrT triads. Since the COs are symbolic, a similar approach is proposed here. There is also a form-content interaction, where the contents take part in the genesis of the form in the first phase (for example, the natural numbers). Therefore, they have a degree of generality that is inferior to the form, which will be stabilized in relation to the interaction in the second phase. In the third stage, the form will be detached from its contents. This is why, when having difficulty in operating with a CO with a certain degree of generality, a CO with a lower degree of generality is used. Similarly, a child in Piaget's concrete operational stage would resort to objects in order to perform operations. This is only natural, since the empirical object in the physical world is an indexical sign assumed by the symbolic object, with the result expressed in the second phase.

In the process of the three phases, it is worth noting that the CO assumes an exogenous character in
the first stage and an *endogenous* character in the third stage. This coincides with the process proposed by Vygotsky in relation to the *external* initial character of the CO, which is a product of the *interaction* between the subject and the environment in a *socio-historical-cultural* process, until its final internalization. Thus, the CO becomes part of the IS in its stabilized third phase.

4. Building conceptual networks

Since we concur with Vygotsky that a concept is a *generalization* in a *system of concepts* which are linked through *relationships of generality*, this system can be conceived as a *network of concepts*. The *basic network* which needs to be considered for a given concept allows its *generalization* through the three phases. This is called *concept generalization structure*. In turn, a concept as such is related to other concepts, some of which result from its definition. Thus, it can be stated that the *concept extends* and will not cease to extend through an infinite open net. The concept defined by its *structure of generalization* can be regarded as a *basic conceptual scheme*. When different concepts in this conceptual network are combined, new conceptual schemes are formed. In turn, these *conceptual schemes* can be combined to produce new concepts, but these will go through the three phases presenting increasing *levels of generality* and a corresponding *semiotic support*. The construction of these networks and conceptual schemes, and their elaboration in the learning process will be the subject of future studies.

5. Conclusions

The correlation between the IaIrT triads and the *categories of firstness, secondness, and thirdness*, leads to a *process* of incorporation-construction of a *conceptual object* that consists of three phases. These phases correspond to *attributes, relationships*, and the CO's own *structure*. Accordingly, from the point of view of semiotics, they correspond to *icon, index*, and *symbol*. In the same way, these phases correspond to the three types of *inferences*: *abductive, inductive*, and *deductive*. The starting point which *motivates, drives, and generates* the process is a *result* understood as a *particular case of manifestation* of the concept (such as a Pythagorean triple). This *result* raises questions that cannot be answered using the *conceptual network* or *conceptual schemes* of the IS. It is constituted by the *contents* and *relationships* or *forms interacting*, which will be constructed in such a way that they will provide an answer to the questions raised. In the first phase, the *contents* and their *attributes* are considered. These will help to determine the *forms* by means of *abduction*. In the second phase, these *forms* interacting with the *contents* reproduce the particular result. This is the phase where the *forms* and the *contents* can be *generalized* in an *inductive* or *completive* way. Thus, they reach the third phase, where the *forms* become *necessary* in relation to the *contents* (which in turn become a *necessary condition* of the *forms*). The *forms* detach themselves from the *contents,
thus enabling the creation of new contents. In this phase, not only is the structure of the constructed relationships determined, but also new conceptual schemes are produced. These schemes are linked to the rest of the conceptual schemes in the IS.

A valuable idea for teachers is to introduce a concept by presenting a problem that contains a result as indicated in this work. It will raise questions that will destabilize the students’ IS and allow the passage through the three phases of conceptual construction. The result should be chosen according to previously acquired conceptual schemes that allow both the identification of the contents and their attributes, and the abduction of the relations involved in that result.

Finally, it is worth noting that the passage through these three phases (i.e., the passage through the IaIrT triad) involves an interaction between forms and contents as in the third version of the process of equilibration proposed by Rolando García (2000). This provides an up-to-date support to the theory proposed in this work.

Bibliography


