Semiotic-Cognitive Theory of Learning

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Abstract: On the basis of Vygotsky's definition of conceptualization (1995), this work links Piaget and García's stages of development through their triads (1984) to the semiotics introduced by Peirce (1974), who classifies signs in three categories which are associated with three inferences: *abduction*, *induction*, and *deduction*. As a result, the construction of knowledge begins with an *abduction* on the first stage, originating from a *result* which is presented (either deliberately or not, when teaching) as a *problem* for the subject and which destabilizes their *Interpretive System*. The *relations* arising from *abductive generalization* are established on the second stage as an *interaction* between *form* and *content.* By *inductive* or *completive generalization*, the *form* becomes *deductive* on the third stage. It is detached from all content, and is incorporated and linked to the rest of the *forms* of the *Interpretive System.* From the semiotic point of view, this implies the following progression: *icon* → *index* → *symbol.*

1. Introduction

The *semiotic-cognitive learning theory*, which is presented in this work, is a theory of knowledge acquisition that involves central aspects of the theories developed by Jean Piaget and Rolando García (Piaget & García, 1984 ; Piaget, 2002; García, 2000), Lev Vygotsky (1995), and Charles Peirce (1974). It combines them in a formulation which, on the basis of Vygotsky's definition of *conceptualization,* states a correspondence among the three stages of cognitive development introduced by Piaget and García, and the three sign classification categories together with the three forms of inference proposed by Peirce (González, 2012). It is strictly focused on learning from the acquisition of hypothetical-deductive thinking, which is assumed as already acquired in secondary and higher education.

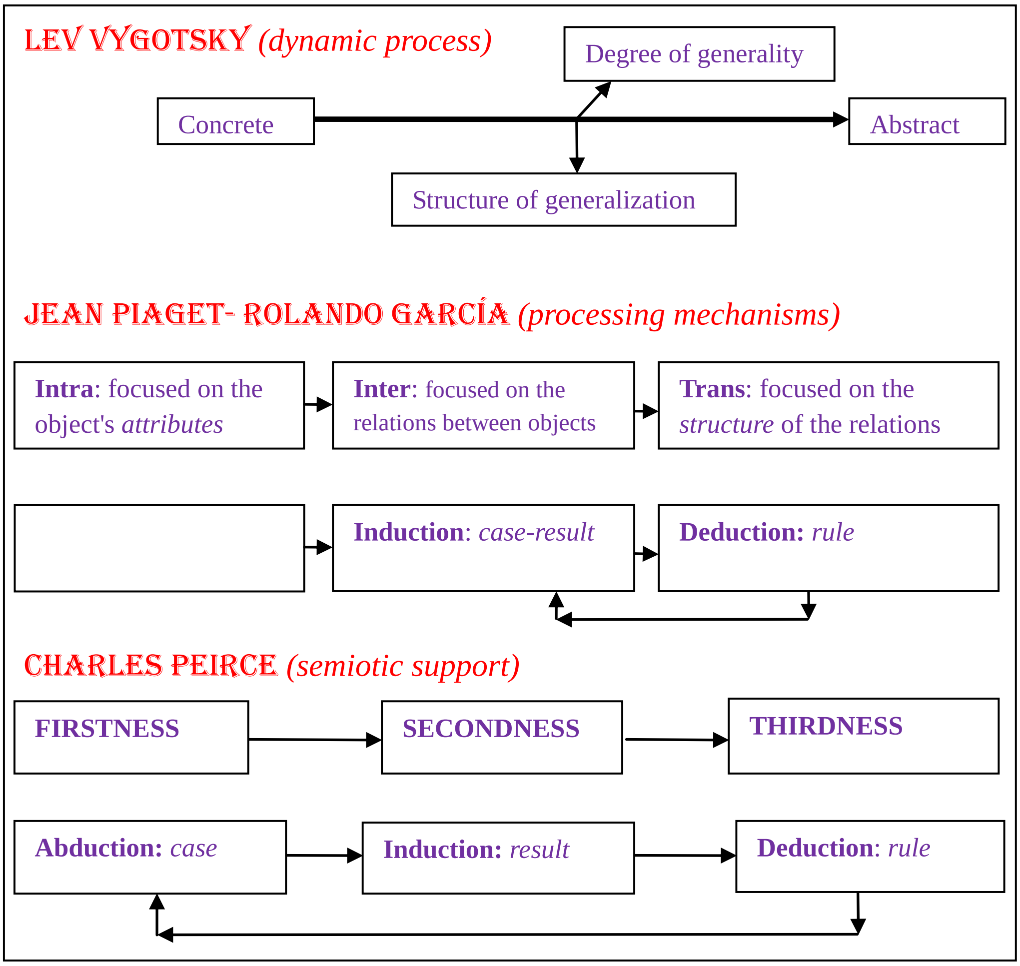
2. Development

Figure 1 shows the main aspects that set the foundations of a *conceptualization process* in the mentioned schools of thought. Vygotsky regards it as a *generalization process* in a *system of concepts* which evolves in a *line of generality*. Along this line, a concept acquires a *degree of generality*. *Relationships of generality* are established between concepts. Conceptual acquisition is a socio-historical-cultural process which first appears in an interpsychological level among people and is then internalized in an intrapsychological level (Vygotsky, 1995).

In *Psychogenesis and the History of Science*, Piaget and García also conceive the idea of conceptualization as a *generalization process.* It is divided in three stages: *intra*, *inter*, and *trans*, as mechanisms that are respectively focused on the *concept attributes* or *conceptual object* (*CO*), the *relations* between concepts, and the *structure* formed by these relations defined as transformations. A typical example given by these authors is the case of plane geometric figures. A triangle, for instance, is defined by its attributes, but it can be regarded as translation and rotation invariant.

These relations or transformations among points of the plane do not alter the distance among the points of the triangle. In turn, they form a group structure. Nevertheless, despite defining three stages, the authors consider only two types of generalization: *inductive* generalization and *completive* (or *constructive*) generalization. *Inductive* generalization consists in generalizing the *result*. Thus, a relation applied to some cases is applied to all of the cases in a determined *conceptual system*. The latter occurs when these *relations* that are associated with a result become *necessary* and the *result* becomes a *necessary condition* for the *relations*. Therefore, given a *case*, the *relations* necessarily lead to the result. In the *trans* stage, the different relations are *coordinated* in the so-called *Interpretive System* (*IS*). In addition, Piaget (García, 2000) states that the *CO* is *assimilated* by the *IS* and transformed by it. In turn, the *IS adjusts* to the *CO* and is simultaneously transformed in a global process of *cognitive equilibrium*.

Fig.1
Diagram of the main components of conceptualization



Vygostky initially considers the sign as an *instrument of conceptualization* and Piaget proposes a *semiotic function* that surpasses the natural language (Radford, 2006). But the *sign* is regarded here as the *bearer of meaning*, i.e., of the *concept*. Taking Vygotsky's theory as a basis, it can be said that in the same way a concept is a generalization in a system of concepts, a *sign* is a *generalization in a system of signs*. *Conceptualizing means generalizing, but also simultaneously defining a system of signs. There is not one without the other.* That is, from the point of view of its meaning, the sign depends on the system of signs it is *interrelated* with. These established relationships between signs are then relationships between concepts, and hence *relationships of generality,* as defined by Vygotsky.

For instance, the concept of integer in the number system and its operations can be defined from the generalization of the concept of subtraction among natural numbers for the cases that cannot be solved in this set (ex. *5*-*9*). The set of integers is also a particular case for the set of rational numbers when the denominator is *1*. Thus, it is a *particular case* of the relationship of generality among integers that defines rational numbers (*a.d=b.c* in *a/b=c/d* with integers *a,b,c,d*). We can see that the signs used with the integers are the ones that will define the signs used with the rational numbers through the relationships of generality that interrelate them. On the other hand, this system of signs implies a *semiotic context* that is given by the *set of relationships of generality* that are established among the *signs* of the system. If, when interpreting these relationships, they are univocally defined (e.g., when they are conventions in a specific field, such as *Z* for integers, *SOS* meaning 'help', traffic lights, etc.), there will be a *syntagmatic context*. If, on the other hand, the relationships of generality that can be established among signs are open to the *interpreter's* different possibilities of interpretation, there will be a *paradigmatic context*. These definitions of *concept* and *sign* imply, therefore, a prior *system of concepts (of signs)* that is necessary to construct them. As Rolando García (2000) proposes, they can be considered *interpreted data*, what he denominates *observable* (a concept that is taken from physics).

Charles Peirce (1974) provides both a *theory of signs* and a *system of correlative inferences*. First, Peirce conceives the *sign* in its aspect of *representamen* as a bearer of a *quality* which stands in place of something else. It represents something – its *object*, a *sign* of real existence, which is in turn interpreted by someone by means of another *sign* denominated *interpretant*. For example, H_2O is an *interpretant* (one of the many that are possible) of the *representamen* 'water' (word) which denotes an *object* that is a *sign* when it is related to other objects (location) and bears the qualities of *colourless, odourless,* and *tasteless liquid*. Any physical object is an *indexical* sign that can only be conceptualized from a system of signs, and therefore requires previous conceptualizations (or signs). Moreover, new conceptualizations will require new signs.

Peirce classifies signs into three categories denominated *firstness, secondness* and *thirdness*. Firstness implies the *quality*, i.e., the *attributes* that are inherent to the object, *abstracting* it from the reference to another object. *Secondness* involves taking the object that bears the attribute in *relation* to another object. In this case, Peirce attributes an *existence* to the object as an *indexical* *sign* (the object that really exists and bears the quality). *Thirdness* is introduced by a *sign*, the *interpretant*, which is a law defined by the relations introduced in secondness. Nevertheless, these concepts are *relative.* For example, an interpretant of a given *degree of generality* can become a representamen in the following stage. Therefore, it is possible to find each of these signs – *representamen*, *object*, and *interpretant* – in all three categories.

The signs defined by Peirce are shown in table 1 (Peirce, 1974; Vitale, 2002; Marafiotti, 2002; Magariños de Morentin, 2008). They are classified by their function. Following what was previously described, we can see that their classification is carried out according to their *degree of generality.* A *qualisign* is a *quality* (for instance, color) embedded in the *sinsign* (traffic light) that expresses a *legisign* (a law: the red light means you cannot cross the street). In turn, an *icon* is a sign (*object*) that is *analogous* to another object (that is, an attribute that is common to the objects, such as a color, or an image that evokes an object, such as a picture, the points in common between two different theories, etc.). An *index* is a sign (*object*) of real existence and contiguity that attracts our attention towards an object (an arrow → among natural numbers: $2 \rightarrow 4$, *2* is assigned *4*). A *symbol* is a sign (*object*) that expresses a level of generality by means of a law (for instance, the variables $n \in N$, $m \in N$, and the expression that is based on them: $m=2n$). A *symbol* is an *object* in the third category of generalization or abstraction, and therefore, an *interpretant*.
A *rheme* is a sign that represents a certain kind of objects (e.g., a flower) and refers to qualities. Thus, it is a *firstness*. A *dicisign* is a *proposition* that involves *rhemes*. It is a *secondness*, so it implies both a *relation* and and *object* of real existence. An *argument* is a *form of reasoning* that involves a *dicisign* as a *premise* and another *dicisign* as a *conclusion*. It is, in essence, an *interpretant*.

Table 1: Peirce's classification of signs

	FIRSTNESS	SECONDNESS	THIRDNESS
REPRESENTAMEN	qualisign	sinsign	legisign
OBJECT	icon	index	symbol
INTERPRETANT	rheme	dicisign	argument

Arrows indicate the direction in which the degree of generality increases.

Nevertheless, this categorization depends on the *level of generality.* An *interpretant* in a determined level of generality can become an *object* in the next level, and even go through all three categories in that same level. Therefore, they constitute fractal relationships which are, as such, also *dialectical* (García, 2000; Piaget, 2002).

An example of a sign going through all three categories in the same level can be found in the natural numbers: their central property is the existence of a *consecutive.* A succession of similar objects (e.g., balls) constitutes an *iconic* representation, and its representamens are *1, 2, 3*,.... We can *anticipate* the need of a *generic representamen*, a *symbol*, which will be a *variable*. But, for example, in order for *m* to become a variable, a previous step is required. Each number must become a possible *result* of the variable by means of an *equivalence relation*, i.e., *m=1, m=2, m=3….* In this case the variable acts as an *index*, since it will *indicate* a specific number. Here, the sign *m* is affected by the *object* (the given natural number) when the equivalence relation is established. This shows, on the other hand, the existence of a *relationship* inherent to *secondness*. It also constitutes an *interaction form* (the *symbol m*) - *content* (the specific number). This relationship applies to all the natural numbers from the equivalence relation of the variable with any natural number. But the generalization of this operation to all the natural numbers *will become a symbol* when representing it by means of a variable becomes a *necessary condition*.

The characteristics of Piaget and García's stages *intra, inter, trans* (*IaIrT*) and the categories of *firstness*, *secondness*, and *thirdness* introduced by Peirce lead to the correspondence *intra*←→ *firstness*, *inter*←→ *secondness*, *trans*←→ *thirdness*, which is proposed in this work. The distribution of signs in the three categories is the *semiotic expression* of the *cognitive mechanisms* (*IaIrT*).

Piaget and Garcia propose two forms of generalization: *inductive* (or *empirical*) generalization and *constructive* (or *completive*) generalization (García, 2000). The former involves an *empirical abstraction* of determined relationships based on attributes *verified* in an empirical object, which, in some cases, if repeated, applies to the set of objects under consideration as well. The latter involves a *reflective abstraction* that projects all the *inferred relationships* in a superior level of coordination that makes them deductive (which means that these relationships must become *necessary*). Piaget and García find a correlation between *inductive generalization* and the *intra* phase because it deals with the object *attributes*, and propose another correlation between *completive generalization* and the *inter* phase, which deals with relations.

In addition to *induction* and *deduction (*which correspond to *firstness* and *secondness),* Peirce's semiotics introduce another type of inference, which he denominates *abduction*. *Abduction* corresponds to *firstness,* which was not considered by Piaget and García, even though Piaget (Piaget & García, 1997; Hernández Ulloa, 2008) mentions it in his later years as an element to be considered. On the other hand, there is a consequence of the correlation between the *IaIrT triads* and the *categories.* With regard to the *attributes* of the *CO*, *abduction* must be taken into account in the situation designated as *cases*. In the second phase, *induction* must be taken into account. Therefore, the first phase is the *contents* phase (in Piagetian terms, *forms* will be constructed in relation to these *contents*) and their *attributes*, which were already constructed in previous stages. The correlation between categories and mechanisms proposed in this work constitutes the base of a clear semiotic expression of these mechanisms, which are represented by their *inferences*. The passage from one phase to another will be achieved by means of *generalizations* (since they result in a succession of abstractions that increase the *degree of generality*). Here, these generalizations will be *abductive*, *inductive* and *completive*. The last two will lead to the third phase, where *deduction* is reached as a third inference. The semiotic definition of these inferences help us better establish the correlations presented and the role of the signs in each of them.

From the semiotic point of view, the definition of these inferences (Fig.1) is founded on the concepts of c*ase*, *result*, and *rule* corresponding to *firstness, secondness, and thirdness*, respectively. The following is a classic example: there are bags containing little balls of different colors (each bag is a *case*), little balls of different colors taken from one of those bags on a table (*result*), bags containing little balls of a same color (*rule*). The *case* involves an *attribute* (color), the *result* involves a *relationship* (between the little balls with certain colors on the table and the bag they were taken from), and the *rule* involves a *structure* (the balls of a *same* color in a *same* bag). Based on these elements, we can state the following (Marafioti, 2002; Vitale, 2002):

Deduction: all the balls in this bag are white (*rule*). These balls were taken from this bag (*case*), therefore (with certainty) these balls are white (*result*). There is a *rule* from which, given a *case*, a *result* can be *inferred*.

Induction: these balls were taken from this bag (*case*), these balls are white (*result*), therefore (probably) all the balls in this bag are white (*rule*). Given a *case* and a *result*, a *rule* can be *inferred*.

Abduction: all the balls in this bag are white (*rule*), these balls are white (*result*), therefore (probably) these balls were taken from this bag (*case*). Given a *rule* and a *result*, a *case* can be *inferred*.

The complete incorporation of a given *CO* requires the passage through the three *stages* or *phases* which are the base of the *cognitive mechanisms*, that is, of the three *inferences* expressed in their corresponding *signs*. In fact, as previously stated, *abduction* is based on *attributes* by means of *iconicity,* since it expresses *analogies* between *different objects*, such as the whiteness of the balls in the bag and the ones on the table. *Induction* is based on *indexicality,* since the *result* is expressed by means of an *indexical relationship* between *objects of real existence*, such as the bags containing balls and the balls on the table (a relationship which is based on attributes, as is 'white').

The conclusion is drawn by means of this *indexical relationship*. Finally, on the basis of these *attributes* and *relationships*, *deduction* is expressed with a *symbol*, since it corresponds to a *general law*. In the above example, the *law* is a *rule* that establishes that all the balls in a given bag are white. Then, perforce, if we take some balls from *that* bag (*case*) and put them on a table, these balls will be white (*result*). Here, the concept of *logical necessity* comes into play.

The passage of the *CO* through the three stages is the *process* that *transforms* its aspects. They change from those of an *iconic sign* to those of a *symbolic sign* when it is incorporated as an *interpretant* in the *Interpretive System*. How is this passage produced? In the first phase, the *cases* refer to the *contents* of the *CO*. These will have determined *attributes* which will, in turn, define a *system of contents* (e.g., the balls in the bags, the system of natural numbers, etc.). *Abduction* requires a *result* that is the *trigger/motivator* of new knowledge and *destabilizes* the *IS*. Peirce regarded this *fact* (*result*) as surprising or exceptional. Nevertheless, if it cannot be incorporated by the *IS*, it will be destabilized. This *specific result* involves determined *contents* defined in this phase and a *relationship* to be revealed as a *hypothesis* by means of *abduction*. The process is initiated with the *genesis* of the *form* developed from the *stabilized IS* (before the *result* destabilizes it) and it will involve the *rule* which, together with the *result*, is part of the definition of *abduction*. Therefore, this *rule* (here, the *hypothesized relationship)* will be in function of the *case* (the *contents attributes*) based on the *result*. Thus, the *case* will be *inferred* by the *rule* and the *result*, as is required by *abduction*. Pythagorean triples are an example of this (González, 2012). When considering the triples of natural numbers *(3, 4, 5)* and *(6, 8, 10)*, where the components fulfill $3^2 + 4^2 = 5^2$ and $6^2 + 8^2 = 10^2$, some questions about the obtention of all the Pythagorean triples of natural numbers arise: how many are there? Which ones are they? How can they be obtained? In this case, the contents are triples of natural numbers and their attributes are those corresponding to the natural numbers and the Pythagorean relationship. Both chosen triples are connected by an *indexical relationship* which is quite easy to notice in this case: *(6, 8, 10)* = 2.*(3, 4, 5)*. This generates the *proportional form* applied to the triples. In order to obtain this *relationship*, it is necessary to *compare* cases. Then, given a *result (6, 8, 10)* and the *proportional form rule*, it is possible to *infer* the *case (3, 4, 5)* and all the other *cases* associated to these triples by resorting to *retroduction* (from the *rule* to the *case*). When a new case is presented, such as the Pythagorean triple *(5, 12, 13)*, which is not proportional to the former triples, the established *rule* cannot be applied, and new *cases* and new *abductions* will be possible and necessary.

The *form* obtained on the basis of *abduction* in the first phase is applied in the second phase to the *cases* that demonstrate and reproduce the *results* establishing the *indexical* relationship. It is usually said that *abduction* explains the *results*. Moreover, this is the phase where *form* and *content* interact. In these conditions and in this stage, the next step is exploring how the *form* can be applied to all the cases that constitute the *system of contents* defined by the *attributes* in the previous stage. If the *form* applied to *some cases* can be applied to *all the cases* responding to similar *attributes*, then we can speak of *inductive generalization*.

In the third phase, the *form* obtained from the *results* related to the *contents* and their *attributes* becomes necessary to them, and they become a *necessary condition* of the *form*. This means that the *results* become *deducible* from the *form* when it is applied to *all the cases*. This *form*, which until now had an *indexical expression* given by its application to *specific cases*, acquires a *symbolic* character and is *detached* from the *content*, becoming a *pure form*. This means that it becomes a part of the *interpretant*, and it will be stabilized when it is coherently linked to the rest of the *forms* of the *IS* which, by incorporating it in this process of *equilibration* by means of *assimilation* and *accommodation*, is extended and transformed into an *IS'*. The following scheme is obtained by passing from the first to the third stage: *CO* → *CO´*, *IS* → *IS´*. In turn, the *structure* of this stage will be given by the *attributes* of the *relationships* involved.

In the example of the Pythagorean triples, the *proportional form* will be noted as $n.(a,b,c)$, with $n \in N$. (a,b,c) is a Pythagorean triple and is clearly a *syntagmatic* and *symbolic form* which has been *detached* from the initial specific contents and can be extended to any Pythagorean triple, even to other *contents*, such as real numbers. That is, as a *form*, its structure would be *(). (, ,)*, which can be applied to any *content* when valid. If the passage from the second to the third phase is produced because the *content attributes* expressed by the *results* are necessary to obtain those *forms*, then a *completive generalization* can take place without the need of an *inductive generalization*. On the other hand, it is important to highlight that these are *dialectic processes*, and therefore these three phases are relative to a certain *degree of generality*. Thus, a phase that is regarded as *trans* in a given *level* can become an *intra* phase in the following *level*. Finally, when the cycle is closed, in addition to new *forms*, *contents* can be extended in the same way number sets are extended.

The semiotic bases of these three inferences show that they are collaborative and almost simultaneously formed. In fact, what we have denominated *rule* depends on the *relations* expressed by means of the *results*. In turn, these *relations* depend on the *cases* determined by the *attributes*. However, this does not mean that the passage through the three stages is simultaneous, since *attributes*, *relations*, and *structures* must be considered in that order. Therefore, there is an *order* of *focus*: by focusing on the *attributes*, *relations* are constructed. These *relations* will constitute the *rule*. By focusing on the *relations*, we can discern the *structures*, and by focusing on the *structures*, we consider the *linkage* of the *forms* in the *IS*.

The complete chart of correlations is as follows:

Table 2: correlations in each semiotic-cognitive phase

	Phase 1	Phase 2	Phase 3
CO situation	*Case*	*Result*	*Rule*
CO focused on	*Attributes*	*Relations*	*Structure*
Category	*Firstness*	*Secondness*	*Thirdness*
Sign	*Icon*	*Index*	*Symbol*
Inference	*Abduction*	*Induction*	*Deduction*

3. Application in the university entrance course

The following is an account of the previously described sequence in the specific case of university entrants.

A field study conducted at Universidad Nacional de General Sarmiento (UNGS) (González, 2012) shows that some of the students in the admission course redefine the *sign* $\sqrt{}$ in a diagnostic activity presented on their first day at university, based on their *IS* at the moment of starting the course. The students are asked to give the result of some basic and compound arithmetic operations using natural numbers. These operations are presented as written expressions ending in an equality sign, which reinforces the idea that a result is being requested. A central objective is to observe how students *interpret* the *signs* involved in these operations. Some of these involve the symbols $\sqrt{4}$ and $\sqrt{5}$. The aim is to observe how they interpret the sign $\sqrt{}$ based on the cases given. The *natural* numbers are the *content* of the *cases* based on their *attributes*. The students are requested to obtain the result of the operations $1+\sqrt{4}=$ and $1+\sqrt{5}=$, where the sign = is an *index* that is associated to the *result*, as previously stated, and induces the student to obtain it. The students who provide an answer, understand the meaning of the sign and try to obtain a result. Since $\sqrt{5}$ is an irrational number, its exact value can only be interpreted *symbolically*. Thus, the second expression can only be solved by means of the identity $1+\sqrt{5}=1+\sqrt{5}$. This is important since it shows that the operation itself is the *exact result*.

In one of the groups of students under study, *12% give* the sign $\sqrt{}$ the *conventional* or *syntagmatic* meaning. They provide the following *results*: $1+\sqrt{4}=3$ and $1+\sqrt{5}=1+\sqrt{5}$. In the passage *attribute→ relations → structure* with a *semiotic support*: i*con → index → symbol*, these students show that they are in the last phase *in relation to the concepts involved*. These students simply *infer* the *results* by applying the *conventional rule* (resulting from a *syntagmatic context*) to the different *cases*. In that group, *21%* of the students use the sign $\sqrt{}$ according to the *conventional rule* for the *cases* of natural numbers that are *perfect squares*. However, they *redefine* the rule in the case of

non-perfect squares: $\sqrt{()} \to \frac{()}{2}$. In the case of 5, the answer provided is $\sqrt{(5)} \to \frac{(5)}{2}$. That is, these students understand $\sqrt{\ }$ as a division by two. This way, their results are $1+\sqrt{4}=3$ and $1+\sqrt{5}=\frac{7}{2}$.
This group is making an *abduction*. In fact, they focus on *attributes* first: the *natural numbers,* discriminating between *perfect squares* and *non-perfect squares*. In the first case, the *conventional rule* is applied. However, as previously stated, $1+\sqrt{5}$ constitutes the *result* itself in the second case. Since the students cannot apply the *conventional rule* to obtain a *result* in the set of natural numbers (or among the numbers they know), they *hypothesize* the *rule* in *function* of the *case*. That is, they *infer* the case from this *rule* and the *result*. They do not consider the *conventional* rule (the *syntagm*) applicable in this *case*. Their *semiotic context* is *paradigmatic*. In this context, the *relationships of generality* are not determined because the students have not acquired the conventional rule for every case. The *relationships of generality* are constructed in function of the possible rules previously incorporated in their *IS*. Moreover, since the *attributes* of the *case* are involved, the search will be *iconic*. That is, by *analogy*, $\sqrt{()} \to \frac{()}{2}$. The first *sign* is *analogous* to a division by 2 (since it is a square root). Therefore, these students are focused on the *iconic* phase. They will be able to continue to the *indexical* phase, but they will fail to reach the last elaboration phase of the *symbol* due to their ambiguous definition in function of the *case*. The last phase, the *deductive* one, will be reached when they are able to understand the fact that the symbol $\sqrt{\ }$ must have an *univocal signification* in relation to *all* the *cases*. This will happen when they cease to depend on the *cases* and the *form* is consequently detached from the *content*.
Rolando García (2000) proposes a third version of the *theory of equilibrium* by Piaget. According to this theory, the constructive process of knowledge results from the interaction of the *form* (logical forms) with the *content* (physical world) by means of the mechanism of the *IaIrT* triads. Since the *CO*s are *symbolic*, a similar approach is proposed here. There is also a *form-content* interaction, where the *contents* take part in the genesis of the *form* in the first phase (for example, the natural numbers). Therefore, they have a *degree of generality* that is inferior to the *form*, which will be stabilized in relation to the interaction in the second phase. In the third stage, the *form* will be *detached* from its *contents*. This is why, when having difficulty in operating with a *CO* with a certain *degree of generality*, a *CO* with a lower degree of generality is used. Similarly, a child in Piaget's concrete operational stage would resort to objects in order to perform operations. This is only natural, since the *empirical object* in the physical world is an *indexical sign* assumed by the *symbolic object*, with the *result* expressed in the second phase.
In the process of the three phases, it is worth noting that the *CO* assumes an *exogenous* character in

the first stage and an *endogenous* character in the third stage. This coincides with the process proposed by Vygotsky in relation to the *external* initial character of the *CO*, which is a product of the *interaction* between the subject and the environment in a *socio-historical-cultural* process, until its *final internalization*. Thus, the *CO* becomes part of the *IS* in its stabilized third phase.

4. Building conceptual networks

Since we concur with Vygotsky that a *concept* is a *generalization* in a *system of concepts* which are linked through *relationships of generality*, this system can be conceived as a *network of concepts*. The *basic network* which needs to be considered for a given *concept* allows its *generalization* through the three phases. This is called *concept generalizaton structure*. In turn, a *concept* as such is related to other *concepts*, some of which result from its definition. Thus, it can be stated that the *concept extends* and will not cease to extend through an infinite *open net*. The concept defined by its *structure of generalization* can be regarded as a *basic conceptual scheme*. When different *concepts* in this *conceptual network* are combined, new *conceptual schemes* are formed. In turn, these *conceptual schemes* can be combined to produce *new concepts*, but these will go through the three phases presenting increasing *levels of generality* and a corresponding *semiotic support*. The construction of these networks and conceptual schemes, and their elaboration in the learning process will be the subject of future studies.

5. Conclusions

The correlation between the *IaIrT* triads and the *categories* of *firstness*, *secondness*, and *thirdness*, leads to a *process* of incorporation-construction of a *conceptual object* that consists of three phases. These phases correspond to *attributes*, *relationships*, and the *CO*'s own *structure*. Accordingly, from the point of view of semiotics, they correspond to *icon*, *index*, and *symbol*. In the same way, these phases correspond to the three types of *inferences*: *abductive, inductive*, and *deductive*. The starting point which *motivates*, *drives*, and *generates* the process is a *result* understood as a *particular case* of *manifestation* of the *concept* (such as a Pythagorean triple). This *result* raises questions that cannot be answered using the *conceptual network* or *conceptual schemes* of the *IS*. It is constituted by the *contents* and *relationships* or *forms interacting*, which will be constructed in such a way that they will provide an answer to the questions raised. In the first phase, the *contents* and their *attributes* are considered. These will help to determine the *forms* by means of *abduction*. In the second phase, these *forms* interacting with the *contents* reproduce the particular result. This is the phase where the *forms* and the *contents* can be *generalized* in an *inductive* or *completive* way. Thus, they reach the third phase, where the *forms* become *necessary* in relation to the *contents* (which in turn become a *necessary condition* of the *forms*). The *forms* detach themselves from the *contents*, thus enabling the creation of *new contents*. In this phase, not only is the *structure* of the *constructed relationships* determined, but also new *conceptual schemes* are produced. These schemes are linked to the rest of the *conceptual schemes* in the *IS*.

A valuable idea for teachers is to introduce a concept by presenting a *problem* that contains a *result* as indicated in this work. It will raise questions that will destabilize the students' *IS* and allow the passage through the three phases of *conceptual construction*. The *result* should be chosen according to previously acquired *conceptual schemes* that allow both the identification of the *contents* and their *attributes*, and the *abduction* of the *relations* involved in that result.

Finally, it is worth noting that the passage through these three phases (i.e., the passage through the *IaIrT* triad) involves an interaction between *forms* and *contents* as in the third version of the *process of equilibration* proposed by Rolando García (2000). This provides an up-to-date support to the theory proposed in this work.

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